Frequency-Dependent Shot Noise as a Probe of Electron-Electron Interaction in Mesoscopic Diffusive Contacts

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The frequency-dependent shot noise in long and narrow mesoscopic diffusive contacts is numerically calculated. The case of arbitrarily strong electron-electron scattering and zero temperature of electrodes is considered. For all voltages, the noise increases with frequency and tends to finite values. These limiting values are larger than the Poissonian noise and increase with voltage nearly as $V^{4/3}$. This allows one to experimentally determine the parameters of electron-electron interaction.

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Basically, electron-electron scattering appears in two different forms. First, this is the decay of quasiparticle states (phase-breaking) and second, this is relaxation of nonequilibrium distribution function of electrons. In last two decades, the equilibrium phase-breaking processes were extensively investigated in relation with weak-localization corrections to conductivity[1] and universal conductance fluctuations.[2,3] In particular, it was found[1] that in dirty metals the disorder strongly enhances electron-electron (e-e) interactions at low temperatures, which even makes questionable the validity of Landau concept of quasiparticles[4] in low-dimensional systems (for a recent review of e-e scattering in mesoscopic systems see paper by Blanter[5]). Generally, the theoretical results are in a reasonable agreement with the experimental data, and this allows determining the phase-breaking time from weak-localization experiments[6].

By far less is known about the kinetics of electron systems in the presence of e-e scattering, which results in relaxation of nonequilibrium distribution of electrons to the Fermian one. While conserving the total energy of electron system, it smooths down the peculiarities of distribution function of electrons. For example, it affects the shape of electron distribution function in a wire placed between two different reservoir electrodes with a finite voltage drop between them. Recently, Pothier *et al.*[7] performed direct measurements of electron distribution function using tunnel superconducting probes and determined the parameters of e-e interaction from their data. However their results appeared to be inconsistent with any of existing theories of e-e scattering.

Although the shape of electron distribution function does not affect the conductivity of metals (except for small quantum corrections), it is crucial in the semiclassical theory of nonequilibrium noise in solids.[8] In particular, it enters into the semiclassical expression for the shot noise in diffusive mesoscopic contacts.[9] Hence this noise may be used for determining the parameters of e-e scattering.

The effects of strong electron-electron scattering on the zero-frequency shot noise were studied within the electron effective temperature approximation[10,11] and were shown to increase the ratio $S_I/2eI$ from $1/3$ to $\sqrt{3}/4$. Though this increase was experimentally observed by Steinbach *et al.*,[12] it is difficult to quantitatively estimate the parameters of e-e scattering from it. The reason is that the zero-frequency noise is determined by the shape of distribution function near the middle of the contact, which becomes Fermian at relatively weak e-e scattering and does not change with its further increase.

The situation is different for the finite-frequency noise in long and narrow contacts with strong external screening, i. e., with a close ground plane[13] or coaxial grounded shielding.[14] It was shown that for noninteracting electrons, the high-frequency shot noise tends to eI if pile-up of charge in the contact is forbidden[13] and to $2eI$ if it is allowed.[14] More recently, Naveh *et al.*[15] obtained that for strong e-e scattering and zero temperature of electrodes, the noise infinitely grows with frequency while remaining linear in current. Below we show that this is not the case: instead of diverging, the actual high-frequency noise tends to a finite value nonlinearly depending on voltage. We also propose a method for quantitative determination of the parameters of e-e scattering from measurements of zero-temperature high-frequency shot noise.

In what follows we use the coaxial model,[14] although all the results also apply to the ground-plane one. Assume that the contact of length L is a cylinder of circular section with a diameter $2r_0 \ll L$ consisting of a dirty metal with conductivity σ (see Fig. 1, the inset) and connects two massive electrodes. The contact is screened from the ambient space by the third perfectly conducting coaxial grounded electrode, which is separated from its surface by a thin insulating film of thickness δ_0 and the dielectric constant ε_d. The external circuit is assumed to

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have a large grounding capacity, which lifts the condition of zero net charge of the contact at finite frequencies.

The spectral density of current noise, e.g., at the left edge of the contact is given by the formula[14]

$$S_I(\omega) = \frac{4}{RL} \int\limits_0^L dx\, K(x,\omega) T_N(x),\qquad (1)$$

where R is the resistance of the contact, x is the longitudinal coordinate, and $T_N(x)$ is the local "noise temperature" determined in terms of the electron distribution function $f(\epsilon, x)$ as follows:

$$T_N(x) = \int d\epsilon\, f(\epsilon, x)[1 - f(\epsilon, x)] \qquad (2)$$

The kernel of the integral (1) is given by the expression

$$K(x,\omega) = 2(\gamma_\omega L)^2$$
$$\times \frac{\cosh[2\gamma_\omega(L-x)] + \cos[2\gamma_\omega(L-x)]}{\cosh(2\gamma_\omega L) - \cos(2\gamma_\omega L)}, \qquad (3)$$

where $\gamma_\omega = (\omega \varepsilon_d / 4\pi\sigma\delta_0 r_0)^{1/2}$. At sufficiently high frequencies, the kernel K exponentially decreases with x. This decrease has a simple physical explanation. At contact dimensions much larger than the Debye screening length, the local current fluctuations inside the contact, which result from the randomness of impurity scattering, induce the current fluctuations at the contact edges through the long-range fluctuations of electrical field. However at finite frequencies, the electric lines of force emerging from the middle points of the contact are intercepted by the screening electrode and do not reach the contact edges. Hence it is only the portions of the contact adjacent to its edges that contribute to the measurable noise. Therefore for calculating the high-frequency noise in such contacts, it is very important to know the exact distribution function of electrons near their edges.

In our semiclassical approach, the distribution function f obeys the diffusion equation

$$D\frac{d^2}{dx^2} f(\epsilon, x) + I_{ee}(\epsilon, x) = 0, \qquad (4)$$

where D is the diffusion coefficient. At zero temperature, the boundary conditions for this equation at the left and the right ends of the contact are

$$f(\epsilon, 0) = \theta(-\epsilon), \quad f(\epsilon, L) = \theta(eV - \epsilon), \qquad (5)$$

where θ is the step function and V is the voltage drop across the contact.

Recently, Eq. (4) was solved using the phenomenological approximation of effective electron temperature,[10,11] i.e. the distribution function was saught in the form

$$f_T(\epsilon, x) = \left[1 + \exp\left(\frac{\epsilon - eVx/L}{T_e(x)}\right)\right]^{-1}, \qquad (6)$$

where T_e was the coordinate-dependent temperature of electron gas. As the collision integral is zero for arbitrary f_T chosen in form (6), Eq. (4) reduces to an energy-balance equation, whose solution at zero temperature of electrodes is

$$T_e = eV\sqrt{3x(L-x)}/\pi L. \qquad (7)$$

As T_e exhibits squire-root singularities at the edges of the contact, substitution of Eq. (7) into (1) results in high-frequency noise diverging[15] as $\omega^{1/4}$. This unphysical divergency is due to the inadequate description of the distribution function near the contact edges by the effective temperature model. Indeed, for $f = f_T$ with T_e given by Eq. (7), the first term of Eq. (4) diverges as $x^{-3/2}$, while the second term remains is zero throughout the length of the contact. Hence f deviates from f_T near the contact edges and the squire-root singularity is smoothed out no matter how strong the e-e scattering.

In this paper, we numerically calculate the high-frequency shot noise for the simplest collision integral with energy-independent transition probabilities:

$$I_{ee}(\epsilon) = -\frac{\lambda_{ee}}{\epsilon_F} \int d\epsilon' \int d\omega$$
$$\times \{f(\epsilon)f(\epsilon'-\omega)[1 - f(\epsilon-\omega)][1 - f(\epsilon')]$$
$$- f(\epsilon-\omega)f(\epsilon')[1 - f(\epsilon)][1 - f(\epsilon'-\omega)]\}. \qquad (8)$$

By doing so, we restrict ourselves to the Landau concept of quasiparticle scattering and disregard the interference between e-e and impurity scattering.[1] This implies that the relevant electron energies are sufficiently high: $\epsilon \gg \tau^{-1}(p_F/k)(p_F l)^{-2}$, where τ is the elastic scattering time, p_F is the Fermi momentum, k is the inverse Debye screening length, and l is the elastic mean free path of electrons.[16] In the gas approximation, where $k \ll p_F$, the dimensionless scattering amplitude equals $\lambda_{ee} = \pi^2 k/64 p_F$. However in realistic metals $k/p_F \sim 1$, and λ_{ee} should be renormalized by corrections of higher order in interaction.

The actual behavior of f near the contact edges may be understood from the following semiquantitative reasoning. Select a point $x_0 \ll L$ near the left edge of the contact. Suppose that $f(x_0) = f_T$ with $T_e(x_0)$ given by Eq. (7) and solve Eq. (4) with boundary conditions $f(\epsilon, 0) = \theta(-\epsilon)$ and $f(\epsilon, x_0) = f_T(\epsilon, x_0)$ in the range $0 < x < x_0$. Because of the divergence of $\partial^2 f_T/\partial x^2$ at $x = 0$ it is reasonable to expect that the diffusion term in Eq. (4) will dominate over I_{ee} at sufficiently small x_0 and the latter may be omitted. Then the resulting diffusion equation is easily solved and retaining only terms linear in x, one obtains for the noise temperature $T_N(x) = (2\ln 2)\, T_e(x_0)x/x_0$. The crossover point x_0 is determined from the condition that the diffusion term in Eq. (4) be of the order of the collision integral, i.e. $D/x_0^2 \sim \lambda_{ee} T_e^2(x_0)/\epsilon_F$. This results in an estimate $x_0 \sim L\alpha^{-1/3}$, where $\alpha = \lambda_{ee}(eVL)^2/\epsilon_F D$ is the dimensionless parameter characterizing the relative strength of

e-e interaction in the contact. From Eq. (1), it follows that the limiting value of high-frequency noise is

$$S_I(\infty) = \frac{2L}{R} \left. \frac{dT_N}{dx} \right|_{x=0} \sim eI\alpha^{1/6}. \qquad (9)$$

The saturation frequency may be determined from the condition $\gamma_\omega \sim x_0^{-1}$, which gives

$$\omega_s \sim \frac{\sigma}{\varepsilon_d} \frac{\delta_0 r_0}{L^2} \alpha^{2/3}.$$

To test these semiqualitative conclusions, Eq. (4) was numerically solved for different values of the dimensionless parameter α using the finite-difference method on a lattice of 100×100 sites. Figure 1 shows the coordinate dependence of the noise temperature T_N calculated for $\alpha = 10$. It is clearly seen that in the middle of the contact, T_N is close to T_e, while it remains almost unperturbed by the e-e interaction near the edges.

Figure 2 shows the frequency dependences of noise for five different values of α ranging from 0 to 10^4. At zero frequency, all the values of noise are located in a narrow range $0.33 < S_I/2eI < 0.43$. As the frequency increases, the lower bound for the noise and (especially) the spacing between the curves also increase. In particular, this implies that at finite frequencies the ratio $S_I/2eI$ is essentially voltage-dependent. The noise reaches saturation for all α considered, but the saturation frequency increases with α. The saturation noise is $2eI$ for $\alpha = 0$ and also increases with α. Note that for the maximum of considered values $\alpha = 10^4$, the zero-frequency shot noise coincides with the result of effective-temperature model to the third decimal place, whereas the high-frequency noise is only nearly three times the Poissonian one. The limiting values of high-frequency noise are plotted versus α in Fig. 3. On a log-log scale, this dependence presents almost a straight line except for the lowest value $\alpha = 10$. At large α, its slope corresponds to $S_I(\infty)/2eI \propto \alpha^{0.1683}$. This exponent is very close to the value $1/6$ that results from the above semiquantitative reasoning. Possibly, the discrepancy could be made even smaller by increasing the number of sites in the lattice and/or α. In any event, the above qualitative consideration provides a reasonably good understanding of the behavior of high-frequency noise.

Naveh *et al.*[15] proposed that the increase of nonequilibrium noise at high frequencies may be used for distinguishing between the cases of strong and weak e-e scattering. Our calculations provide a basis for quantitative estimates of the parameters of e-e scattering. By measuring the voltage dependence of saturated high-frequency noise and determining the corresponding exponent, one may test the validity of Landau theory for this case. Knowing

the diffusion coefficient and ϵ_F, it is also possible to determine the parameter of e-e interaction λ_{ee} from these data. In these measurements, the voltage must be sufficiently high to exclude the effects of interference between e-e and impurity scattering and to avoid quantum noise dominating over the shot one.[17] Because of slow growth of the noise-to-current ratio with voltage, proper care should be taken to eliminate heating effects.

In summary, we have shown that in the presence of e-e scattering, the shot noise in long diffusive contacts increases with frequency and tends to a finite value. This value is larger than $2eI$ and increases with voltage nearly according to the law $S_I(\infty)/2eI \propto V^{1/3}$.

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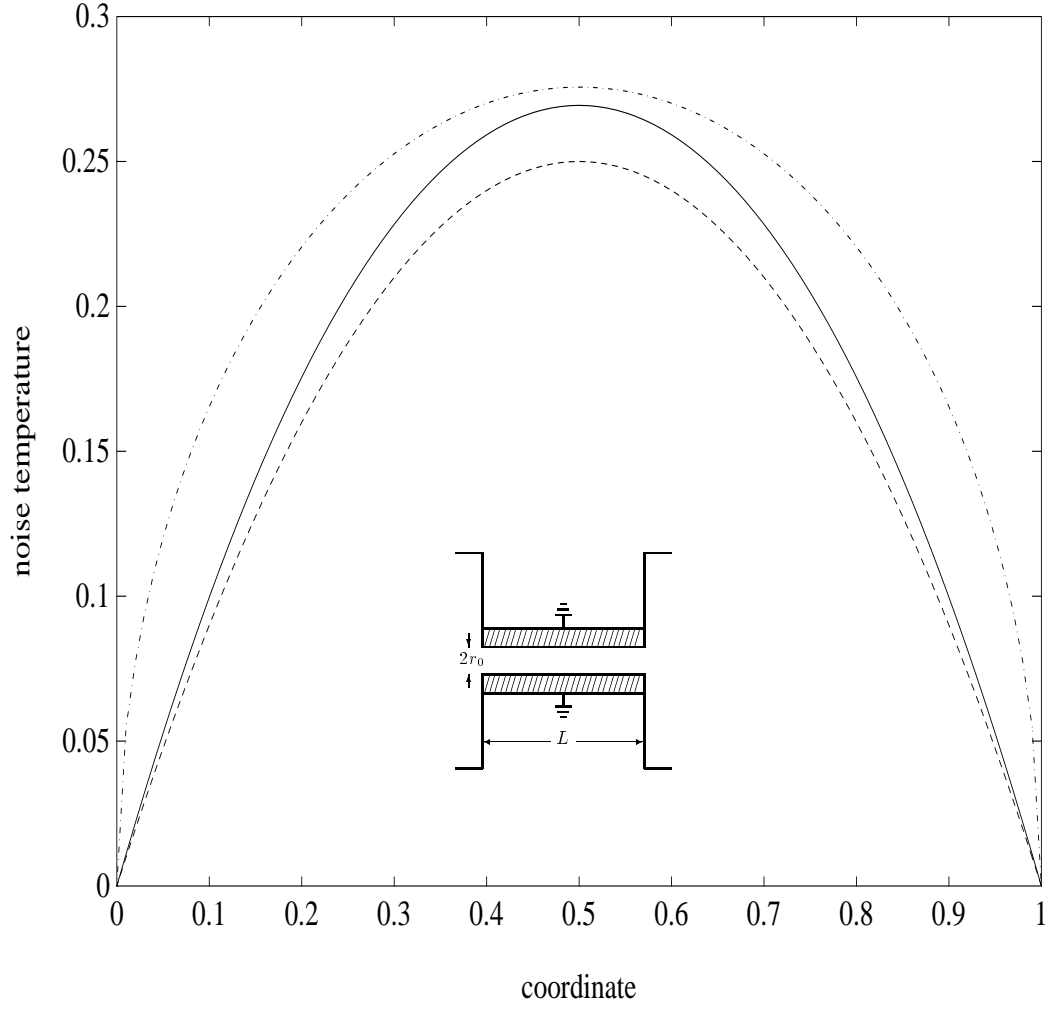
FIG. 1. Dependence of noise temperature T_N/eV on coordinate x/L for $\alpha = 10$. The dashed line shows T_N for noninteracting electrons, and the dash-dot line shows T_e calculated from the energy-balance equation. Inset shows the longitudinal section of the shielded contact.
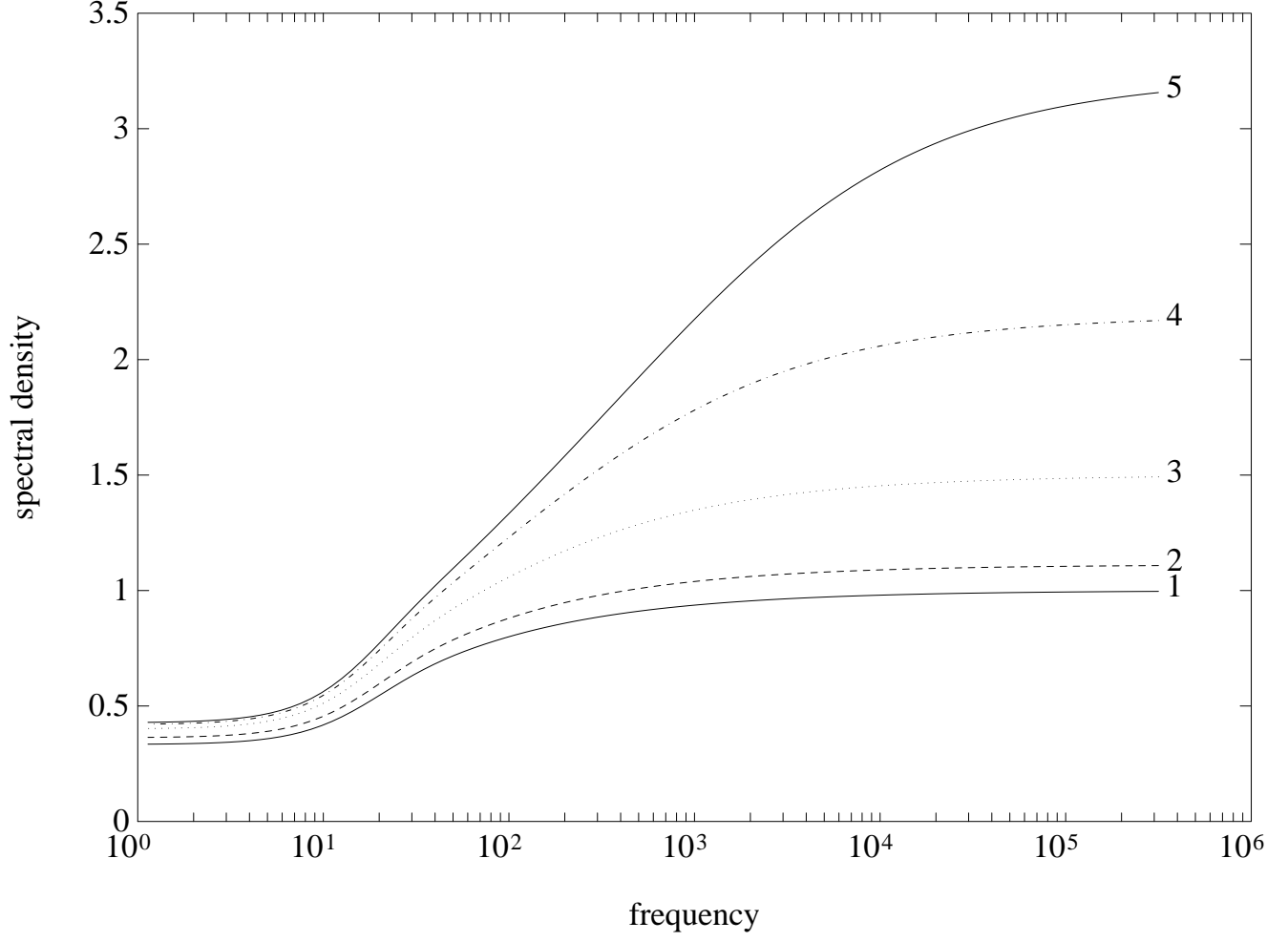
FIG. 2.　Dependences of normalized spectral density $S_I/2eI$ on dimensionless frequency $\omega\varepsilon_d/4\pi\sigma\delta_0 r_0$ for (1) $\alpha = 0$, (2) $\alpha = 10$, (3) $\alpha = 100$, (4) $\alpha = 1000$, and (5) $\alpha = 10000$.
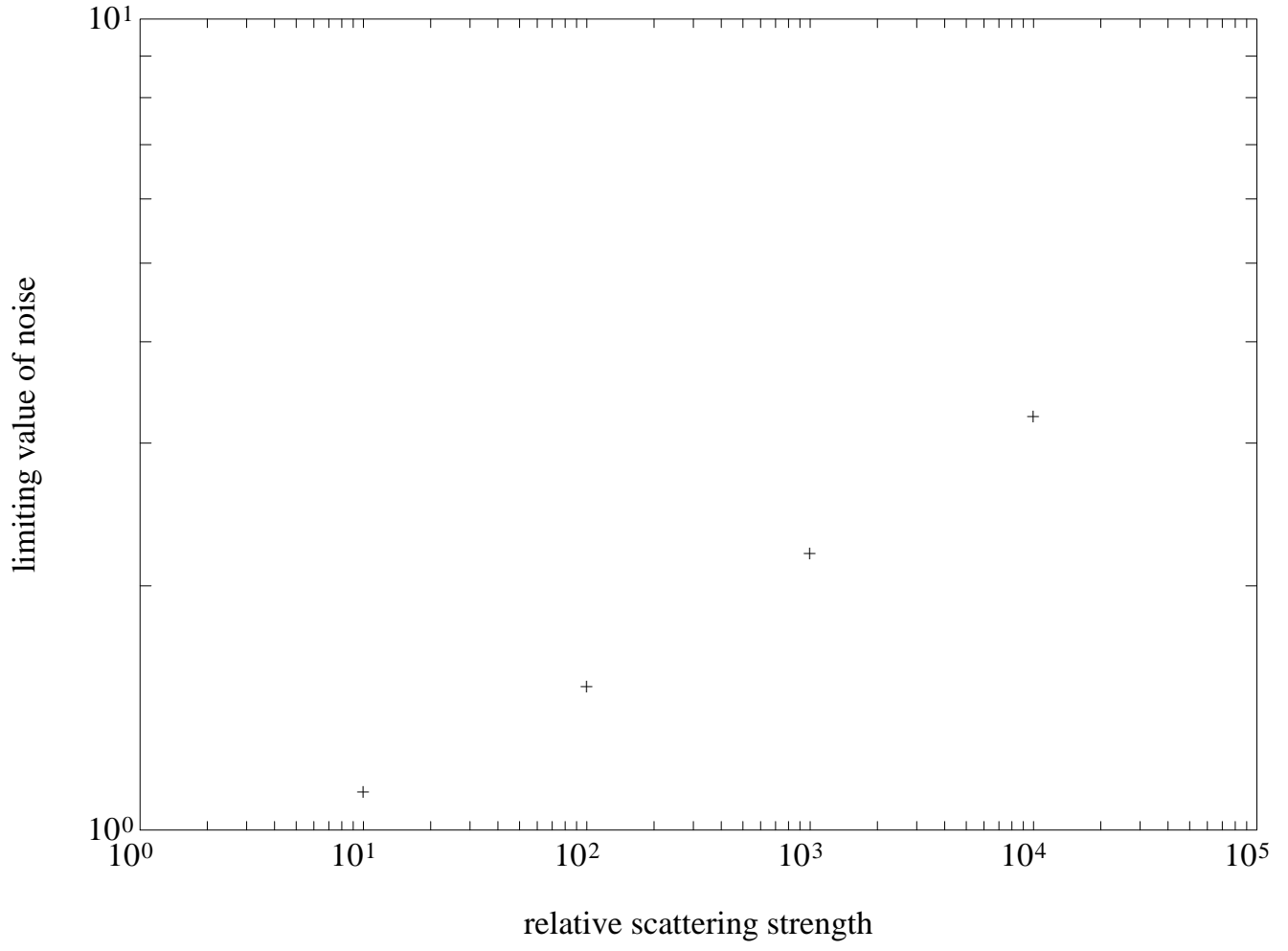
FIG. 3. Log-log plot of saturated high-frequency noise $S_I(\infty)/2eI$ vs. electron-electron scattering parameter α.